Standards for Mathematical Practice

The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important "processes and proficiencies" with longstanding importance in mathematics education. The first of these are the NCTM process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council's report *Adding It Up*: adaptive reasoning , strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently and appropriately) and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy).

1. **Make sense of problems and persevere in solving them.**
   Mathematically proficient students:
   - explain to themselves the meaning of a problem and looking for entry points to its solution.
   - analyze givens, constraints, relationships, and goals.
   - make conjectures about the form and meaning of the solution attempt.
   - consider analogous problems, and try special cases and simpler forms of the original problem.
   - monitor and evaluate their progress and change course if necessary.
   - transform algebraic expressions or change the viewing window on their graphing calculator to get information.
   - explain correspondences between equations, verbal descriptions, tables, and graphs.
   - draw diagrams of important features and relationships, graph data, and search for regularity or trends.
   - use concrete objects or pictures to help conceptualize and solve a problem.
   - check their answers to problems using a different method.
   - ask themselves, "Does this make sense?"
   - understand the approaches of others to solving complex problems.

2. **Reason abstractly and quantitatively.**
   Mathematically proficient students:
   - make sense of quantities and their relationships in problem situations.
     - ✓ *decontextualize* (abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents and
     - ✓ *contextualize* (pause as needed during the manipulation process in order to probe into the referents for the symbols involved).
   - use quantitative reasoning that entails creating a coherent representation of quantities, not just how to compute them
   - know and flexibly use different properties of operations and objects.

3. **Construct viable arguments and critique the reasoning of others.**
   Mathematically proficient students:
   - understand and use stated assumptions, definitions, and previously established results in constructing arguments.
   - make conjectures and build a logical progression of statements to explore the truth of their conjectures.
   - analyze situations by breaking them into cases
   - recognize and use counterexamples.
   - justify their conclusions, communicate them to others, and respond to the arguments of others.
   - reason inductively about data, making plausible arguments that take into account the context
   - compare the effectiveness of plausible arguments
   - distinguish correct logic or reasoning from that which is flawed
     - ✓ elementary students construct arguments using objects, drawings, diagrams, and actions.
     - ✓ later students learn to determine domains to which an argument applies.
   - listen or read the arguments of others, decide whether they make sense, and ask useful questions

4. Model with mathematics.

Mathematically proficient students:

- apply the mathematics they know to solve problems arising in everyday life, society, and the workplace.
  - ✓ In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community.
  - ✓ By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another.
- simplify a complicated situation, realizing that these may need revision later.
- identify important quantities in a practical situation
- map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas.
- analyze those relationships mathematically to draw conclusions.
- interpret their mathematical results in the context of the situation.
- reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

5. Use appropriate tools strategically.

Mathematically proficient students

- consider available tools when solving a mathematical problem.
- are familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools
- detect possible errors by using estimations and other mathematical knowledge.
- know that technology can enable them to visualize the results of varying assumptions, and explore consequences.
- identify relevant mathematical resources and use them to pose or solve problems.
- use technological tools to explore and deepen their understanding of concepts.

6. Attend to precision.

Mathematically proficient students:

- try to communicate precisely to others.
- use clear definitions in discussion with others and in their own reasoning.
- state the meaning of the symbols they choose, including using the equal sign consistently and appropriately.
- specify units of measure and label axes to clarify the correspondence with quantities in a problem.
- calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the context.
  - ✓ In the elementary grades, students give carefully formulated explanations to each other.
  - ✓ In high school, students have learned to examine claims and make explicit use of definitions.

7. Look for and make use of structure.

Mathematically proficient students:

- look closely to discern a pattern or structure.
  - ✓ Young students might notice that three and seven more is the same amount as seven and three more.
  - ✓ Later, students will see 7×8 equals the well-remembered $7 \times 5 + 7 \times 3$, in preparation for the distributive property.
  - ✓ In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$.
- step back for an overview and can shift perspective.
- see complicated things, such as some algebraic expressions, as single objects or composed of several objects.

8. Look for and express regularity in repeated reasoning.

Mathematically proficient students:

- notice if calculations are repeated
- look both for general methods and for shortcuts.
- maintain oversight of the process, while attending to the details.
- continually evaluate the reasonableness of intermediate results.

Standards for Mathematical Practice in Action

Practice	Sample Student Evidence	Sample Teacher Actions
1. Make sense of problems and persevere in solving them	☐ Display sense-making behaviors ☐ Show patience and listen to others ☐ Turn and talk for first steps and/or generate solution plan ☐ Analyze information in problems ☐ Use and recall multiple strategies ☐ Self-evaluate and redirect ☐ Assess reasonableness of process and answer	☐ Provide open-ended problems ☐ Ask probing questions ☐ Probe student responses ☐ Promote and value discourse ☐ Promote collaboration ☐ Model and accept multiple approaches
2. Reason abstractly and quantitatively	☐ Represent abstract and contextual situations symbolically ☐ Interpret problems logically in context ☐ Estimate for reasonableness ☐ Make connections including real life situations ☐ Create and use multiple representations ☐ Visualize problems ☐ Put symbolic problems into context	☐ Model context to symbol and symbol to context ☐ Create problems such as “what word problem will this equation solve?” ☐ Give real world situations ☐ Offer authentic performance tasks ☐ Place less emphasis on the answer ☐ Value invented strategies ☐ Think Aloud
3. Construct viable arguments and critique the reasoning of others	☐ Listen, respond, and question others ☐ Use examples and non-examples ☐ Support beliefs and challenges with mathematical evidence ☐ Form logical arguments with conjectures and counterexamples ☐ Use multiple representations for evidence ☐ Use precise mathematical vocabulary	☐ Create a safe and collaborative environment ☐ Model respectful discourse behaviors ☐ “Find the error” problems ☐ Promote student to student discourse (do not mediate discussion) ☐ Plan effective questions or Socratic formats ☐ Provide time and value discourse
4. Model with mathematics	☐ Connect math (numbers & symbols) to real-life situations ☐ Symbolize real-world problems with math ☐ Make sense of mathematics ☐ Apply prior knowledge to solve problems ☐ Choose and apply representations, manipulatives and other models to solve problems ☐ Use strategies to make problems simpler ☐ Use estimation and logic to check reasonableness of an answer	☐ Model reasoning skills ☐ Provide meaningful, real world, authentic performance-based tasks ☐ Make appropriate tools available ☐ Model various modeling techniques ☐ Accept and value multiple approaches and representations
5. Use appropriate tools strategically	☐ Choose appropriate tool(s) for a given problem ☐ Use technology to deepen understanding ☐ Identify and locate resources ☐ Defend mathematically choice of tool	☐ Provide a “toolbox” at all times with all available tools – students then choose as needed ☐ Model tool use, especially technology for understanding
6. Attend to precision	☐ Communicate (oral and written) with precise vocabulary ☐ Carefully formulate questions and explanations (not retelling steps) ☐ Decode and interpret meaning of symbols ☐ Pay attention to units, labeling, scale, etc. ☐ Calculate accurately and effectively ☐ Express answers within context when appropriate	☐ Model problem solving strategies ☐ Give explicit and precise instruction ☐ Ask probing questions ☐ Use ELA strategies of decoding, comprehending, and text-to-self connections for interpretation of symbolic and contextual math problems ☐ Guided inquiry
7. Look for and make use of structure	☐ Look for, identify, and interpret patterns and structures ☐ Make connections to skills and strategies previously learned to solve new problems and tasks ☐ Breakdown complex problems into simpler and more manageable chunks ☐ Use multiple representations for quantities ☐ View complicated quantities as both a single object or a composition of objects	☐ Let students explore and explain patterns ☐ Use open-ended questioning ☐ Prompt students to make connections and choose problems that foster connections ☐ Ask for multiple interpretations of quantities
8. Look for and express regularity in repeated reasoning	☐ Design and state “shortcuts” ☐ Generate “rules” from repeated reasoning or practice (e.g. integer operations) ☐ Evaluate the reasonableness of intermediate steps ☐ Make generalizations	☐ Provide tasks that allow students to generalize ☐ Don’t teach steps or rules, but allow students to explore and generalize in order to discover and formalize ☐ Ask deliberate questions ☐ Create strategic and purposeful check-in points